Looking at Spacings to Assess Streakiness

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September 2013
The Problem

- Collect hitting data for all players in a season.
- Focus on patterns of hot and cold hitting for all players.
- Is there evidence that players are truly streaky?
- Or maybe players are not truly streaky ...
- Patterns we see are similar to the patterns one would observe by flipping a collection of coins.
Binary Sequence

- Observe sequence of hitting data for a particular baseball player.
- For each opportunity, observe a hit (1) or a out (0).
- Interested in the pattern of streaks and slumps
- Often interested in the *spacings*, the number of failures between consecutive successes.
- The media reports *large* values of these spacings.
- In 2012 season, Josh Reddick had a “0 for 30” streak.
Observe the following data for a player in 15 at-bats:

0 0 1 1 1 0 0 0 0 1 0 1 0 0 1

Spacings are 2, 0, 0, 4, 1, 2

The values of these spacings are informative about the player’s success probabilities.
How to Measure Streakiness?

- Long runs of successes or long runs of failures.
- Permutation test – measure deviation from randomness by a p value
- Focus here is on a Bayes test statistic – compare two models
A Geometric Model

- Let \( y_1, \ldots, y_n \) denote observed spacings for a particular player.
- Assume \( \{y_i\} \) independent, where \( y_j \) is Geometric (\( p_j \)):
  \[
  f(y_j|p_j) = p_j(1 - p_j)^{y_j}, \quad y_j = 0, 1, 2, \ldots
  \]
- Talk about *true* streakiness by considering models on probabilities \( p_1, \ldots, p_n \).
The Consistent Model: Model $M$

- If hitter is *not* streaky or truly consistent, one believes hitting probabilities are constant:

  $$p_1 = \ldots = p_n = p.$$ 

- Assume we have little information about location of $p$.
- Assign $p$ a noninformative prior

  $$g(p) = \frac{1}{p(1-p)}.$$
The Streaky Model: Model $M_K$

- If hitter is truly streaky, one believes hitting probabilities vary across the season.
- Assume $p_1, \ldots, p_n$ is distributed according to a beta density
  \[ g(p) \propto p^{a-1}(1-p)^{b-1}, \quad 0 < p < 1. \]
- Parameterize in terms of mean and precision
  \[ \eta = \frac{a}{a+b}, \quad K = a + b. \]
- Fix value of $K$ and assign $\eta$ noninformative prior
  \[ g(\eta) = \frac{1}{\eta(1-\eta)}. \]
A Test Statistic

- A Bayes factor.
- Ratio of the marginal probabilities of the observed data $y$ under the two models $M_K$ and $M$:
- Bayes factor in support of true streakiness is

$$BF_K = \frac{f(y|M_K)}{f(y|M)}.$$  

Values of $BF_K > 1$ support the streaky model, and values $BF_K < 1$ support the consistent model.
Use a Subjective Approach for Obtaining $K$

- Think about the variability of success probabilities $\{p_j\}$ if the player was truly streaky.
- Specify a standard deviation of the probabilities.
- This can be used to assess a value of $K$. 
Look at All Players in the 2012 Season

- Consider all players with at least 200 at-bats. (Want to exclude pitchers.)
- For each player, use Bayes factor with $\log K = 5$ to test for streakiness.
- Measure Bayes factor on log scale (positive values of $\log BF_K$ support streakiness)
- Plot log Bayes factors against the sample size (number of at-bats).
Bayes Factors for all 2012 Players

![Bayes Factors Scatter Plot]

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Label 2 Interesting Outliers

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Tally spacings - get \{y, f_y\}
Plot $\log f_j$ (vertical) against $j$ (horizontal)
If geometric, plot will be linear
Graph shows deviations from geometric
Geometric Plot for Reddick

\[
\log(nk) \quad \text{vs} \quad k
\]
Geometric Plot for Headley
Evidence for streakiness ($\log BF > 0$) for 120 (38%) of the players.

Observe a very streaky player (Reddick) and a very consistent hitter (Headley)

But maybe players are truly consistent and we are observing streakiness due to multiplicity.

Are these streaky outcomes a result of a “reasonable” consistent model for hitting for all players?
Observe data \( \{(y_j, n_j)\} \), where \( y_j \) is the number of hits in \( n_j \) at-bats for \( j \)th player.

Assume \( y_j \) is binomial(\( n_j, P_j \)) (consistent model)

Assume success probabilities \( P_1, \ldots, P_N \) are exchangeable.

Posterior estimate of \( P_j \) shrinks (adjusts) observed batting average \( y_j/n_j \) towards the overall average.
What Streaky Data Do We See from a Consistent Model?

- Estimate hitting probabilities $P_1, \ldots, P_N$ from the exchangeable model.
- Simulate individual at-bat results $\{w_1, \ldots, w_{n_j}\}$ from independent Bernoulli distributions with constant probability $\hat{P}_j$.
- For each sequence of hitting data, compute Bayes test in support of streakiness $\log BF_K$.
- Find the number of players where there is support for streakiness ($\log BF_K > 0$).
- Repeat this process 200 times.
Predictive Distribution of Number of Streaky Players

Looking at Spacings to Assess Streakiness
Consistent model (one hitting probability for each player) seems useful in predicting pattern of streakiness of all players in a season.

Players may exhibit streakiness for a particular season, but little evidence to suggest that players have streaky tendencies

End of story?
But There are Other Definitions of “Success”

- Different definitions of batting “success”, such as “hit”, “strikeout”, “home run”.
- Strikeout and home run rates are more informative about the player abilities.
- See this by fitting a random effects model.
A Random Effects Model

- Observe success rates $y_1/n_1$, ..., $y_N/n_N$ of $N$ players.
- Give $p_1, ..., p_N$ a distribution $g(p)$ (random effects distribution).
- Estimate $g(p)$ from the data.
- Learn about the fraction of the variability in the rates that is due to (1) variation between player abilities (the $p_j$), and (2) binomial variation (luck).
- On this scale, strikeout rates exceed home run rates which exceed batting averages.
Look at 53 Seasons of Batting Data

- Looked at batting sequences for all players with at least 200 AB for seasons 1960 through 2012
- Considered three definitions of “success”, strikeout, home run, and hit
- For each season, performed the same predictive analysis – simulated predictive distribution of the number of streaky hitters \((\log BF > 0)\) assuming consistent model
- Focused on predictive p-value of observed number of streaky hitters
Histogram of Predictive P-Values for H, HR, SO
For home run rates and strikeout rates, there is more observed season streakiness among players that one would predict based on a consistent model.

There is a long right-tail in the streakiness distribution.

So there are “interesting” patterns of streakiness.

Focus on home run hitting – do there exist streaky home run hitters?
What Were the Streakiest Seasons of Home Run Hitting?

- What do these streaky seasons look like?
- Focus on the 1998 Ivan Rodriguez
Graph of 1998 Ivan’s Home Runs
Are There Players Who Tend to Be Streaky?

- Suppose one is “streaky” if $\log LBF > 0.5$
- Focus on the players who had the most streaky seasons
- Carl Yastrzemski had six streaky seasons in his career (1961 - 1983)
What Were the Streakiest Hitters Among the Sluggers (40+ HR)?

- 2002 Shawn Green, 2009 Albert Pujols, 1962 Willie Mays
- Focus on the 2002 Shawn Green
Geometric Plot for Green

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What Was the Most Consistent Hitter/Season Among the Sluggers (40+ HR)?

- 1963 Hank Aaron
- In fact, for practically all of Aaron’s seasons, \( \log BF < 0 \)
- Aaron was a remarkably consistent home run hitter
- Was it due to his style of hitting?
1963 Hank Aaron
Great slugger for the Phillies

Wrote a paper some years ago exploring timing of his 548 home runs

Looked at spacings

Mike was streaky early, but more consistent pattern of home run hitting later in his career

Confirmed his patterns using this approach
Closing Comments

- Bayes factor is a useful measure of streakiness
- Posterior predictive analysis helpful for seeing if there is “interesting” streakiness
- Hit/out is a different story than HR or Strikeout sequences
- Some players have interesting patterns of streakiness or consistency
The Great Streak in 1941

- 56-game hitting streak
- Recently, Retrosheet has some play-by-play records available for 1941 season
- Obtain DiMaggio’s sequence of hits and outs for all at-bats
- Is there evidence that Joe was streaky in the 1941 season?
Was Joe Streaky in ’41?

- No ... log $BF \approx 0$
- No evidence to support streaky or consistent model
- What does this mean?
- Actually, great hitters are unlikely to be streaky during a season


R package BayesTestSpacings, http://bayes.bgsu.edu/spacings (download package and look at examples)